

String Processing / Counting / Automata

Problem Statement:

Count the number of strings of length n over the alphabet $\{a, b, c\}$ that do not contain the substring "ab".

Return the count modulo $10^9 + 7$.

Example:

Input:

$n = 2$

Output:

8

Explanation:

All length-2 strings from $\{a, b, c\} = 9$.

The only invalid one is "ab".

So, $9 - 1 = 8$.

Brute Force:

- Generate all strings of length n .
- Check each if it contains "ab".
- Count valid strings.
- Time complexity: $O(3^n)$, impractical for large n .

Optimal Approach (DP with Automata):

- Use DP to count strings without "ab".
- Keep track of last character to avoid "ab".
- States:
 - $dp[i][lastChar]$: number of valid strings of length i ending with $lastChar$.
- Transitions based on $lastChar$ and new character.

Java Code:

```
public class CountStringsNoAB {  
    static final int MOD = 1_000_000_007;
```

```

public static int countStrings(int n) {
    // Map characters to indices: a=0, b=1, c=2
    // dp[i][last] = number of strings length i ending with lastChar
    long[][][] dp = new long[n + 1][3][3];

    // Base case: strings of length 1
    for (int ch = 0; ch < 3; ch++) {
        dp[1][ch] = 1;
    }

    for (int i = 2; i <= n; i++) {
        for (int last = 0; last < 3; last++) {
            for (int curr = 0; curr < 3; curr++) {
                // Avoid substring "ab" = Last='a'(0), curr='b'(1)
                if (last == 0 && curr == 1) continue;
                dp[i][curr] = (dp[i][curr] + dp[i - 1][last]) % MOD;
            }
        }
    }

    long result = 0;
    for (int ch = 0; ch < 3; ch++) {
        result = (result + dp[n][ch]) % MOD;
    }
    return (int) result;
}

public static void main(String[] args) {
    int n = 2;
    System.out.println(countStrings(n)); // Output: 8
}

```

Output: